* One fct must be a polynomial

* Products that one function is not the derivative of the other

What you'll Learn About

- How to integrate a product by that cannot be done by recognition

Proof of Integration by Parts

1. *Find* $\frac{d}{dx}(uv)=$

$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

2. Integrate both sides

$\int \frac{d}{dx}(uv) = \int u\frac{dv}{dx} + \int v\frac{du}{dx}$

$uv = \int u\frac{dv}{dx} + \int v\frac{du}{dx}$

3. Solve for $\int u\frac{dv}{dx}$

$\int u\frac{dv}{dx} = uv - \int v\frac{du}{dx}$

$\boxed{\int u\,dv = uv - \int v\,du}$

$\int xe^x$

① Derivative of left side

② Antiderivative of right side

Use ultra violet minus super vdu to integrate the following

2. $\int xe^x dx = xe^x - \int 1e^x dx = \boxed{xe^x - e^x + C}$

$\int u\,dv = uv - \int v\,du$

$u = x \qquad dv = e^x dx$

$\frac{du}{dx} = 1 \qquad v = e^x$

$du = 1dx$

Use tabular integration to integrate the following

2. $\int xe^x dx = xe^x - \int 1e^x dx = \boxed{xe^x - e^x + C}$

$\int x e^x dx$

x	e^x
1	e^x

$\int x e^x$

x	e^x
1	e^x
0	e^x

Use tabular integration to integrate the following

$\int x^2 e^{-x} dx =$ $x^2 \mid e^{-x}$ $2x \mid -e^{-x}$ $2 \mid e^{-x}$ $0 \mid -e^{-x}$	6. $\int x^2 e^{-x} = -x^2e^{-x} - \int -2xe^{-x}dx$ $2x \mid -e^{-x}$ $= -x^2e^{-x} + \int 2xe^{-x}dx$ $2 \mid -e^{-x}$ $= -x^2e^{-x} + \left[-2xe^{-x} - \int -2e^{-x}dx\right]$ $= -x^2e^{-x} - 2xe^{-x} + \int 2e^{-x}dx$ $= \boxed{-x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C}$ 8. $\int x^2 \cos\left(\frac{x}{2}\right) = \boxed{2x^2\sin\left(\frac{x}{2}\right) + 8x\cos\left(\frac{x}{2}\right) - 16\sin\left(\frac{x}{2}\right) + C}$ $2x \mid 2\sin\left(\frac{x}{2}\right)$ $2 \mid -4\cos\left(\frac{x}{2}\right)$ $0 \mid -8\sin\left(\frac{x}{2}\right)$ Solve the initial value problem using tabular integration

Solve the initial value problem

	11. $\int \frac{dy}{dx} = \int (x+2)\sin x \quad y = 2 \text{ and } x = 0$ $\begin{array}{c\|c} (x+2) & \sin x \\ \hline 1 & -\cos x \\ \hline 0 & -\sin x \end{array}$ $y = -(x+2)\cos x + \sin x + C$ $2 = -2 + C$ $4 = C$ $\boxed{y = -(x+2)\cos x + \sin x + 4}$ 16. $\int \frac{dy}{dx} = \int 2x\sqrt{x+2} \quad y(-1) = 0$ $y = \int 2x (x+2)^{1/2} dx$ $\begin{array}{c\|c} 2x & (x+2)^{1/2} \\ \hline 2 & \frac{2}{3}(x+2)^{3/2} \\ \hline 0 & \frac{4}{15}(x+2)^{5/2} \end{array}$ $y = \frac{4x}{3}(x+2)^{3/2} - \frac{8}{15}(x+2)^{5/2} + C$ $0 = -\frac{4(5)}{3(5)} - \frac{8}{15} + C$ $\boxed{y = \frac{4x}{3}(x+2)^{3/2} - \frac{8}{15}(x+2)^{5/2} + \frac{28}{15}}$

 $C = \frac{28}{15}$ $\quad 0 = -\frac{20}{15} - \frac{8}{15} + C$

Use tabular integration to integrate the following

10. $\int x^2 \ln x dx$

x^2	$\ln x dx$
$2x$	$x \ln x - x$
2	?
0	

$\int \ln x$	$x^2 dx$
$\frac{1}{x}$	$\frac{1}{3}x^3$

$$\int x^2 \ln x dx = \frac{1}{3}x^3 \ln x - \int \frac{1}{x} \cdot \frac{x^3}{3} dx$$

$$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 dx$$

$$\boxed{= \frac{1}{3}x^3 \ln x - \frac{1}{3} \cdot \frac{1}{3}x^3 + C}$$

Use ultra violet minus super vdu to integrate the following

~~10. $\int x^2 \ln x dx$~~

$\int \ln x$	dx
$\frac{1}{x}$	x

$$= x \ln x - \int 1 dx$$

$$= \boxed{x \ln x - x + C}$$

	Use tabular integration to integrate the following A. $\int \arcsin(x)\,dx = x\arcsin x - \int x(1-x^2)^{-1/2}$ $\frac{1}{\sqrt{1-x^2}}$ x $= \boxed{x\arcsin x + (1-x^2)^{1/2} + C}$ 19. $\int e^x \cos(2x)\,dx$